

Title: Can You Can a Can?

A discovery/exploration lesson investigating the production of a cylindrical can of a given volume with the least amount of material, thus minimizing the cost of production.

Links to Outcomes:

- **Problem Solving** Students will use problem solving to investigate the possibilities for dimensions of a cylindrical can, given its volume.
- **Communication** Students will discuss the geometric concepts of area and volume and present conclusions in a written report.
- **Reasoning** Students will gather data, make conjectures, and build arguments based on the interpretation of a graph.
- **Connections** Students will make connections between algebra and geometry, in a real-world situation.
- **Measurement** Students will use measurement to obtain data which will be used to draw conclusions.
- **Geometry** Students will use the geometric properties of a figure to write an equation for a mathematical model.
- **Algebra** Students will represent a real-world situation as an equation and reach conclusions from interpreting its graph.
- **Technology** Students will use a computer software package and/or a graphics calculator in their investigation.
- **Cooperation** Students will demonstrate the ability to investigate mathematics in groups of two or three.

Brief Overview:

Why is a Campbell's soup can shaped as it is? Why not pack soup in a tuna can? In this lesson, the student will investigate volume and surface area of a can to determine the dimensions of the "perfect" can - i.e., a can which requires the least amount of material for a given volume. Data will be collected to compare several sizes of cans - is the can a "perfect can?" If not, why?

Grade/Level:

Grades 8-12: Geometry through Calculus

Duration/Length:

This lesson is expected to take 1 to 1½ days, depending on the ability and/or level of the class.

Prerequisite Knowledge:

Students should be familiar with formulas to find area of a circle, surface area of a cylinder, and volume of a cylinder. Students should be skilled in using Derive and/or the TI-82 calculator.

Objectives:

Students will:

- measure height and circumference of a cylindrical can.
- relate circumference of a circle to radius in order to find the radius of the can.
- use the formula for volume of a cylinder to compute the volume of a can.
- use the formulas for surface area and volume of a cylinder to write an equation which models surface area as a function of dimensions of the cylinder.
- use Derive and/or the TI-82 calculator to make computations and graph model.
- use the graph to determine if the can is a “perfect” can.
- investigate the rationale for a can’s not being “perfect.”
- pool data and make conjectures about the dimensions of the cans found on the grocers’ shelves.

Materials/ Resources/ Printed Materials:

Computers, Derive™, TI-82 calculators
Canned goods
Tape measure
Worksheets

Development/Procedures:

Students should be instructed to bring to class a canned food item the day the activity is presented. Tell students that, whether large or small, cans they bring must be “regular” cylindrical cans (i.e. cans with perfectly straight sides and a flat top and bottom). On the day the activity is to be presented, the teacher should bring two cans of roughly equal volume (not weight): a short, squat can (like tuna fish comes in) and a taller can (a soup can, for example). The teacher should determine the radius and height of the two cans. **Activity 1** is a guided demonstration led by the teacher. **Activity 2** is a related assignment intended to be completed by the student, working individually or with a partner. **Activity 3** is an extension that investigates the rationale for the size and shape of a particular can.

Activity 1: Investigating the relationship between dimensions and surface area for a cylinder of given volume.

1. **Background:** The teacher should ask the class to imagine that they are going into business selling their incredibly delicious beef stew. The beef stew will be sold in a can holding 342 cm^3 (the amount found in a typical soup can), and the question has come up relating to the shape of the can: should a short, squat tuna fish type can be used, a taller soup-type can, or does it make any difference which shape can is used, as long as the cans hold the same amount? Whichever can is chosen, a primary concern will be to keep the cost of the can as low as possible. What is an obvious factor in determining the cost of the can to be used? *[Answer: the amount of material used to make each can]* How can we determine how much material is required to make a can? *[Answer: if the thickness of the material is disregarded, the amount of material can be approximated by calculating the total surface area of the can].*

2. For a cylindrical can of radius r and height h , write an equation for the total surface area.

$$[Answer: a = 2(\pi r^2) + 2\pi rh]$$

3. Tell the students the radius and height of the two cans and have them calculate the total surface area for both. Although both cans have the same volume, is the amount of material used to make each can (i.e. the surface area) the same? *[Answer: no]* Which of the two cans should the company use to minimize cost? *[Answer: the taller can]*

4. Might there be some other can of different dimensions (but same volume) that would be an even better choice? Ideally, what we would like to find is the dimensions of the “perfect can,” i.e. the can with given volume requiring the least amount of material to construct. To explore this, we need to go back to our equation for the total surface area, $a = 2(\pi r^2) + 2\pi rh$. In this relation, area is a function of what? *[Answer: radius and height]* Before we can do anything with this problem, we will need to express area as a function of only one variable. We can do this by substituting. What is the formula for the volume of a cylinder?

$$[Answer: v = \pi r^2 h]$$

Solve this relation for h *[Answer: $h = v/(\pi r^2)$]* Now, substitute this expression for h into the area function and simplify.

$$\begin{aligned} [Answer: \quad a &= 2\pi r^2 + 2\pi r(v/(\pi r^2)) \\ a &= 2\pi r^2 + 2v/r \\ \text{since } v &= 342 \text{ in this case,} \\ a &= 2\pi r^2 + 684/r] \end{aligned}$$

5. What does the last equation in the above step represent? *[Answer: For any can containing 342 cm³, the equation gives the surface area for a given radius.]* Now, how can we find the “perfect sized” can? Trial and error using various values for r ? What might be a faster and easier way to check for the smallest surface area? *[Answer: analyze the graph of this equation]*
6. Use Derive™ to graph $a = 2\pi r^2 + 684/r$
7. Interpret the graph (i.e. domain, range, shape, etc.)
8. Use Derive™ to find the coordinates of the minimum point on the graph. What is the significance of these numbers? *[Answer: The abscissa is the value of the radius of the can (containing 342 cm³) which requires the least material to construct. The ordinate specifies this minimum value for surface area.]* Were either of the cans the “perfect” can? *[Answer: yes, the taller can]*

Activity 2: Student analysis of a can

Purpose: To determine if the can that the student brought is a “perfect” can, i.e., if the can was constructed using the minimum material for the volume it contains.

Name _____

Date _____

The Perfect Can

1. Measure the height of your can. Record your measurement.

height: _____

2. Measure the circumference of your can.

circumference: _____

Using the relationship $c = 2\pi r$, determine the radius of your can. Show your work below.

radius: _____

3. Write an expression for the total surface area a of your can in terms of its radius r and height h .

$a =$

4. Using the formula for the volume of a cylinder, solve for h and substitute for h in the equation for total surface area obtained in the previous step.

$a =$

5. Calculate the volume of your can. Show your work.

$v =$

Notice that now you have the surface area of the can expressed solely as a function of r , the radius of your can.

6. Use Derive™ to graph the equation for total surface area obtained in the previous step. Sketch the graph below.

State the domain and range for the function graphed. Explain the meaning of the graph... what does it tell the observer?

domain: _____

range: _____

Explanation:

7. What point on the graph represents information about the "perfect" can?

coordinates: _____

What attributes do the coordinates describe?

abscissa: _____, value: _____

ordinate: _____, value: _____

8. Based on your measurement of the radius of your can, is it a "perfect" can?
9. If your answer in the previous step is "no," what are some possible reasons why a company would use a presumably more expensive "non-perfect" can?

Evaluation:

For **Activity 2**, the teacher will circulate around the room to ensure that students are on task and to answer any questions about the activity. Student worksheets for this activity will be collected and assessed.

Extension/Follow Up:

- Students will investigate why cans are sized and shaped as they are by contacting the manufacturer. (**Activity 3**)
- Students will go to a grocery store and make a list of dimensions and volumes of the cans in one area, say canned fruits. Other students will go to other food sections. Pool data and make conjectures about why cans are produced as they are. Perform statistical analysis appropriate to the course level (i.e., frequency distribution, deviation from the perfect can, etc.).

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